

# Gödel-type universes in Palatini $f(R)$ gravity

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We examine the question as to whether the Palatini  $f(R)$  gravity theories permit space-times in which the causality is violated. We show that every perfect-fluid Gödel-type solution of Palatini  $f(R)$  gravity with density  $\rho$  and pressure  $p$  that satisfy the weak energy condition  $\rho + p \geq 0$  is necessarily isometric to the Gödel geometry, demonstrating therefore that these theories present causal anomalies in the form of closed time-like curves. This result extends a theorem on Gödel-type models to the framework of Palatini  $f(R)$  gravity theory. We concretely examine the Gödel-type perfect-fluid solutions in specific  $f(R) = R - \alpha/R^n$  Palatini gravity theory, where the free parameters  $\alpha$  and  $n$  have been recently constrained by observational data. We show that for positive matter density and for  $\alpha$  and  $n$  within the interval permitted by the observations, this theory does not admit the Gödel geometry as a perfect-fluid solution of its field equations. In this sense, this theory remedies the causal pathology in the form of closed time-like curves which is allowed in general relativity. We derive an expression for a critical radius  $r_c$  (beyond which the causality is violated) for an arbitrary Palatini  $f(R)$  theory. The expression makes apparent that the violation of causality depends on the form of  $f(R)$  and on the matter content components. We also examine the violation of causality of Gödel-type by considering a single scalar field as the matter content. For this source we show that Palatini  $f(R)$  gravity gives rise to a unique Gödel-type solution with no violation of causality.

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## I. INTRODUCTION

The  $f(R)$  gravity theory provides an alternative way to explain the current cosmic acceleration with no need of invoking either a dark energy component or the existence of an extra spatial dimension. The freedom in the choice of different functional forms of  $f(R)$ , however, gives rise to the problem of how to constrain on theoretical and/or observational grounds, the many possible  $f(R)$  gravity theories. A great deal of effort has gone into the study of some features of these theories [1] (see also Refs. [2] for recent reviews). This includes solar system tests [3], Newtonian limit [4], gravitational stability [5] and singularities [6]. General principles such as the so-called energy conditions have also been used to place constraints on  $f(R)$  theory [7]. Recently, observational constraints from several cosmological data sets have also been employed to test the viability of some  $f(R)$  theories [8–14].

In dealing with  $f(R)$  gravity theories two different variation approaches may be followed, namely the metric and the Palatini formalisms. In the metric approach the connection is assumed to be the Levi-Civita connection, and variation of the action is taken with respect to the metric, whereas in the Palatini approach the metric and the affine connections are treated as independent fields and the variation of the action is taken with respect to both

metric and connections. Although these approaches lead to the same set of field equations in the context of general relativity (GR), for a general  $f(R)$  with non-linear term in the Einstein-Hilbert action they give rise to different field equations. In this paper we shall focus on  $f(R)$  gravity in the Palatini formalism.

In both versions of the  $f(R)$  gravity theories the causal structure of four-dimensional space-time has locally the same qualitative nature as that of the flat space-time of special relativity — causality holds locally. The nonlocal question, however, is left open, and violation of causality can occur. However, if gravity is governed by a  $f(R)$  theory instead of GR, various issues of both observational and theoretical nature ought to be reexamined in the  $f(R)$  gravity framework, including the question as to whether these theories permit space-time solutions of their field equations in which the causality is violated.

In general relativity there are solutions to the field equations that have causal anomalies in the form of closed time-like curves. The renowned Gödel model [15] is the best known example of such a solution, which makes apparent that the GR does not exclude the existence of solutions with closed timelike world lines, despite its Lorentzian character that leads to the local validity of the causality principle. The Gödel model is a solution of Einstein's equations with cosmological constant  $\Lambda$  for dust of density  $\rho$ , but it can also be interpreted as perfect-fluid solution (with pressure  $p = \rho$ ) without cosmological constant. In this regard, we recall that it was shown by Bampi and Zordan [16] that every Gödel-type solution of Einstein's equations with a perfect-fluid energy-

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momentum tensor is necessarily isometric to the Gödel spacetime.

Owing to its unexpected properties, Gödel's model has a well-recognized importance and has motivated a number of investigations on rotating Gödel-type models as well as on causal anomalies not only in the context of general relativity (see, e.g. Refs. [17]) but also in the framework of other theories of gravitation (see, for example, Refs. [18]).

In a recent paper, we have examined Gödel-type models and the violation of causality problem for  $f(R)$  gravity in the *metric* variational approach [19], generalizing therefore the results of Refs. [20] and [21]. In this article, to proceed further with the investigation of Gödel-type universes along with the question of breakdown of causality in  $f(R)$  gravity, we extend the results of Refs. [19–21] by examining the question as to whether the  $f(R)$  gravity theories in the *Palatini* formalism admit Gödel-type space-times solutions in which violation of causality can occur for a physically well-motivated matter source. In this way, we extend the results of Ref. [19, 21] in four different regards. First, we demonstrate that every perfect-fluid Gödel-type solution of any Palatini  $f(R)$  gravity with density  $\rho$  and pressure  $p$  and satisfying the weak energy condition  $\rho + p \geq 0$  or equivalently  $df/dR > 0^1$  is necessarily isometric to the Gödel geometry. This extends to the context of Palatini  $f(R)$  the so-called Bampi-Zordan theorem [16] which was established in the context of Einstein's theory, and has been extended recently to the framework of  $f(R)$  in the *metric* formalism [19]. Second, we examine the dependence of the critical radius  $r_c$  (beyond which the causality is violated) with both the Palatini  $f(R)$  gravity and the fluid components ( $p$  and  $\rho$ ) and derive an expression for  $r_c$  that holds for any Palatini  $f(R)$  gravity theory. Third, we concretely illustrate our general results for perfect-fluid Gödel-type solutions in Palatini  $f(R)$  gravity by taking the specific  $f(R) = R - \alpha/R^n$  theory, where the free parameters  $\alpha$  and  $n$  have been recently constrained by a diverse set of observational data. We show that for positive matter density and for  $\alpha$  and  $n$  within the interval permitted by the observational data, this theory does not admit Gödel geometry as a perfect-fluid solution of its field equations. In this sense, this theory remedies the causal pathology in the form of closed time-like curves which is allowed in general relativity. Fourth, we examine the violation of causality of Gödel type by considering a scalar field as a matter source. For this source we show that Palatini  $f(R)$  gravity gives rise to a unique Gödel-type solution with no violation of causality.

## II. $f(R)$ GRAVITY IN THE PALATINI APPROACH

The causality problem in  $f(R)$  gravity theories can be seen as having three interrelated physically determinant ingredients, namely the gravity theory, the space-time geometry and the matter source. Regarding the first ingredient we recall that the action that defines a  $f(R)$  gravity is given by

$$S = \int d^4x \sqrt{-g} \left[ \frac{f(R)}{2\kappa^2} + \mathcal{L}_m \right], \quad (1)$$

where  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$ ,  $f(R)$  is a function of the Ricci scalar  $R$ ,  $\kappa^2 = 8\pi G$ , and  $\mathcal{L}_m$  is the Lagrangian density for the matter fields. Treating the metric and the connection as independent fields, the variation of this action with respect to the metric gives the field equations

$$f_R R_{(\mu\nu)} - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (2)$$

where  $f_R = df/dR$ ,  $T_{\mu\nu} = -(2/\sqrt{-g}) \delta(\sqrt{-g}\mathcal{L}_m)/\delta g^{\mu\nu}$  is the matter energy-momentum tensor, and  $R_{\mu\nu}$  is given in the usual way in terms of the connection  $\Gamma_{\mu\nu}^\rho$  and its derivatives.

The variation of the action (1) with respect to the connections field yields

$$\tilde{\nabla}_\beta (f_R \sqrt{-g} g^{\mu\nu}) = 0, \quad (3)$$

where  $\tilde{\nabla}_\beta$  denotes the covariant derivative associated with the  $\Gamma_{\mu\nu}^\rho$ . If one defines a metric  $h_{\mu\nu} = f_R(R) g_{\mu\nu}$  it can be easily shown that Eq. (3) determines a Levi-Civita connection of  $h_{\mu\nu}$ , which in turn can be rewritten in terms of  $g_{\mu\nu}$  and its Levi-Civita connection in the form

$$\Gamma_{\mu\nu}^\rho = \{\rho_{\mu\nu}\} + \frac{1}{2f_R} (\delta_\mu^\rho \partial_\nu + \delta_\nu^\rho \partial_\mu - g_{\mu\nu} g^{\rho\sigma} \partial_\sigma) f_R. \quad (4)$$

An important constraint, often used to simplify the field equations, comes from the trace of equation (2), which is given by

$$f_R R(\Gamma) - 2f = \kappa^2 T, \quad (5)$$

where  $T = g^{\mu\nu} T_{\mu\nu}$  is the trace of the energy-momentum tensor and  $R(\Gamma) = g^{\mu\nu} R_{\mu\nu}$  is calculated with the connection  $\Gamma_{\mu\nu}^\rho$  given by Eq. (4).

In practice, it turns out to be useful to express the field equations (2) in terms of the metric  $g_{\mu\nu}$ , its derivatives, and the matter fields. To this end, one uses equations (4) and (5) to eliminate the connection  $\Gamma_{\mu\nu}^\rho$  from the field equations (2). After some manipulations one obtains

$$f_R G_{\mu\nu} = \kappa^2 T_{\mu\nu} - \frac{1}{2} (\kappa^2 T + f) g_{\mu\nu} + H_{\mu\nu} f_R - \frac{3}{2f_R} \left[ \nabla_\mu f_R \nabla_\nu f_R - \frac{1}{2} g_{\mu\nu} (\nabla f_R)^2 \right], \quad (6)$$

<sup>1</sup> In the metric formalism, this condition is necessary to ensure that the effective Newton constant  $G_{\text{eff}} = G/f_R$  does not change its sign. At a quantum level, it prevents the graviton from becoming ghostlike.

where  $\nabla_\mu$  denotes the covariant derivative associated with the Levi-Civita connection of the metric  $g_{\mu\nu}$ ,  $\square = g^{\alpha\beta} \nabla_\alpha \nabla_\beta$ ,  $H_{\mu\nu} \equiv \nabla_\mu \nabla_\nu - g_{\mu\nu} \square$ , and  $G_{\mu\nu} = R_{\mu\nu} - R/2 g_{\mu\nu}$  is the Einstein tensor, which is also calculated with the metric Levi-Civita connection.

Having given an account of the first basic ingredient of the causality problem, i.e.  $f(R)$  gravity in the Palatini approach, in the next section we shall examine the second relevant component of this problem, which is the Gödel-type geometries, and discuss how the violation of causality can occur in Gödel-type spacetimes.

### III. GÖDEL-TYPE GEOMETRIES

The Gödel-type class of geometries that we focus our attention on in this article is given, in cylindrical coordinates  $(r, \phi, z)$ , by [21]

$$ds^2 = [dt + H(r)d\phi]^2 - D^2(r)d\phi^2 - dr^2 - dz^2, \quad (7)$$

where

$$H(r) = \frac{4\omega}{m^2} \sinh^2\left(\frac{mr}{2}\right), \quad (8)$$

$$D(r) = \frac{1}{m} \sinh(mr), \quad (9)$$

where  $\omega$  and  $m$  are constant parameters such that  $\omega^2 > 0$  and  $-\infty \leq m^2 \leq +\infty$ . Clearly, for  $m^2 = -\mu^2 < 0$  the metric functions  $H(r)$  and  $D(r)$  become circular functions  $H(r) = (4\omega/\mu^2) \sin^2(\mu r/2)$  and  $D(r) = \mu^{-1} \sin(\mu r)$ , while in the limiting case  $m = 0$  they become  $H = \omega r^2$  and  $D = r$ .

All Gödel-type geometries are characterized by the two parameters  $m$  and  $\omega$ . In this way, identical pairs  $(m^2, \omega^2)$  specify isometric spacetimes [21–23]. Gödel solution is a particular case of the  $0 < m^2 < +\infty$  class of spacetimes in which  $m^2 = 2\omega^2$ .

In order to examine the causality features of Gödel-type we first note that the Gödel-type line element (7) can be rewritten as

$$ds^2 = dt^2 + 2H(r)dt d\phi - dr^2 - G(r)d\phi^2 - dz^2, \quad (10)$$

where  $G(r) = D^2 - H^2$ . In this form it is clear that the circles defined by  $t, z, r = \text{const}$ , are closed time-like curves depending on the sign of  $G(r)$ . Thus, for  $G(r) < 0$  for a certain range of  $r$  ( $r_1 < r < r_2$ , say) the so-called *Gödel circles* defined by  $t, z, r = \text{const}$  are closed time-like curves. By using this inequality along with the equations (8) and (9) it is easy to show that the causality features of the Gödel-type space-times depend upon the two independent parameters  $m$  and  $\omega$  as it follows [21]. For  $m = 0$  there is a critical radius,  $r_c = 1/\omega$ , such that for all  $r > r_c$  there are noncausal Gödel circles defined by  $t, z, r = \text{const}$ . For  $m^2 = -\mu^2 < 0$  the functions  $H$  and  $D$  are trigonometric functions and there is an infinite sequence of alternating causal and noncausal  $t, z, r = \text{const}$  regions without and with Gödel circles. For

$0 < m^2 < 4\omega^2$  noncausal Gödel circles occur for  $r > r_c$  such that

$$\sinh^2 \frac{mr_c}{2} = \left[ \frac{4\omega^2}{m^2} - 1 \right]^{-1}. \quad (11)$$

When  $m^2 = 4\omega^2$  the critical radius  $r_c \rightarrow \infty$ . Thus, for  $m^2 \geq 4\omega^2$  there are no Gödel circles, and hence the breakdown of causality of Gödel-type is avoided.

To close this section, we note that the presence of a single closed timelike curve as, for example, a Gödel's circle, is an unequivocal manifestation of violation of causality. However, a space-time may admit noncausal closed curves other than Gödel's circles. In this paper, by noncausal and causal solutions we mean, respectively, solutions with and without violation of causality of Gödel-type, i.e., with and without Gödel's circles. Clearly this type of violation of causality is not of trivial topological nature, which are obtained by topological identification [24].

### IV. GÖDEL-TYPE SOLUTIONS IN PALATINI $f(R)$ GRAVITY

The third important ingredient in the above mentioned causality problem is the matter source, which we shall discuss in this section. To this end, we first show how the field equations (6) can be greatly simplified for Gödel-type geometries, and then we discuss the role played by two matter sources in the breakdown of causality of Gödel-type.

#### A. Field equations

From Eqs. (7), (8) and (9) it is straightforward to show that the Ricci scalar for the Gödel-type metrics takes a constant value  $R = 2(m^2 - \omega^2)$ . Hence, the last three terms of the field equations (6) vanish. A further simplification comes about if instead of using coordinates basis one uses the following locally Lorentzian basis:

$$\theta^0 = dt + H(r)d\phi, \quad \theta^1 = dr, \quad (12)$$

$$\theta^2 = D(r)d\phi, \quad \theta^3 = dz, \quad (13)$$

relative to which the Gödel-type line element (7) clearly takes the form

$$ds^2 = \eta_{AB} \theta^A \theta^B = (\theta^0)^2 - (\theta^1)^2 - (\theta^2)^2 - (\theta^3)^2. \quad (14)$$

In this basis the field equations (6) reduce to

$$f_R G_{AB} = \kappa^2 T_{AB} - \frac{1}{2} (\kappa^2 T + f) \eta_{AB}, \quad (15)$$

where the nonvanishing components of the Einstein tensor  $G_{AB}$  take the quite simple form

$$G_{00} = 3\omega^2 - m^2, \quad G_{11} = G_{22} = \omega^2, \quad G_{33} = m^2 - \omega^2. \quad (16)$$

In the next subsections we examine whether these theories permit causal and non-causal solutions for two physically well-motivated matter sources, namely a perfect fluid and a single scalar field.

### B. Perfect fluid solutions

We first consider a perfect-fluid of density  $\rho$  and pressure  $p$ , whose energy-momentum tensor in the basis (12) and (13) is clearly given by

$$T_{AB}^{(M)} = (\rho + p) u_A u_B - p \eta_{AB}. \quad (17)$$

Taking into account Eq. (16) and Eq. (15), for this matter source the field equations reduce to

$$2\omega^2 f_R - f = \kappa^2 (\rho - p), \quad (18)$$

$$2(m^2 - \omega^2) f_R - f = \kappa^2 (\rho - p), \quad (19)$$

$$2(3\omega^2 - m^2) f_R + f = \kappa^2 (\rho + 3p). \quad (20)$$

From Eqs. (18) and (19) we obtain  $f_R (m^2 - 2\omega^2) = 0$ . Thus, for  $f(R)$  theories that satisfy the weak energy condition  $f_R > 0$  (see next paragraph for details) we have  $m^2 = 2\omega^2$ , which according to Sec. III defines the Gödel metric. Thus, a general class of perfect fluid Gödel-type solutions of Palatini  $f(R)$  gravity is given by

$$m^2 = 2\omega^2, \quad (21)$$

$$\kappa^2 \rho = m^2 f_R - \frac{f}{2}, \quad (22)$$

$$\kappa^2 p = \frac{f}{2}, \quad (23)$$

where  $f$  and  $f_R$  are evaluated at  $R = 2(m^2 - \omega^2) = m^2$ .

Now, recalling the weak energy condition (WEC) [25] takes the form  $\rho \geq 0$  and  $\rho + p \geq 0$ , it is clear from (22) and (23) that  $m^2 f_R = \kappa^2 (\rho + p)$ , and thus the second WEC inequality is identically satisfied for any Palatini  $f(R)$  gravity theories with  $f_R > 0$ . In this way, equations (21), (22) and (23) show that the Gödel geometry arises as perfect fluid solution of any Palatini  $f(R)$  gravity in which  $\rho + p > 0$ . This result can be looked upon as an extension of Bampi and Zordan [16] theorem<sup>3</sup> to the context of Palatini  $f(R)$  gravity in the sense that for arbitrary  $\rho$  and  $p$  (with  $\rho + p > 0$ ) all perfect-fluid Gödel-type solution of every Palatini  $f(R)$  gravity, which satisfies the condition  $f_R > 0$ , are necessarily isometric to the Gödel geometry.

Regarding the causality properties of this general family of perfect-fluid Gödel-type solutions, we first note that

since they are isometric to Gödel geometry they admit noncausal Gödel circles of radius greater than the critical radius  $r_c$  given by Eq. (11). But, taking into account Eqs. (22) and (23) we have now that

$$r_c = 2 \sqrt{\frac{f_R}{\kappa^2 (\rho + p)}} \sinh^{-1}(1), \quad (24)$$

making apparent that the critical radius, beyond which there exist noncausal Gödel circles, depends on the two physically relevant ingredients, namely the gravity theory and the matter source components, as one would expect from the outset.

Before proceeding, some words of clarification are in order concerning the first inequality of the WEC, which ensures the positivity of the matter density  $\rho$ . In general relativity [ $f(R) = R$ ] Eqs. (23) and (22) clearly yield  $\kappa \rho = \kappa^2 p = m^2/2$ , making clear that both the matter density and the pressure are positive. However, for a general  $f(R)$  (with non-linear terms in  $R$ ) these equations do not necessarily lead to  $\rho > 0$  for all values  $m^2 = 2\omega^2$ . In this way, the above general result concerning perfect-fluid Gödel-type solutions may not hold for some  $f(R)$  gravity if one further demands the first WEC inequality ( $\rho > 0$ ), which from Eq. (22) leads to

$$m^2 f_R - \frac{f}{2} \geq 0, \quad (25)$$

where  $f_R \neq 0$  and both  $f$  and  $f_R$  are evaluated at  $R = m^2$ .

As a concrete example, we consider the extensively discussed  $f(R)$  theory given by

$$f(R) = R - \frac{\alpha}{R^n}, \quad (26)$$

where  $\alpha$  and  $n$  are free parameters to be determined by local gravity constraints and cosmological observations. In the metric approach the gravity theories of the form (26) are known to be plagued with problems [3–5, 27]. In the Palatini approach, however, combination of a dynamical autonomous systems analysis (study of the fixed points and stabilities against perturbations) yields  $n > -1$  for  $\alpha > 0$ , which can be shown to permit cosmological models with radiation-dominated, matter-dominated and de Sitter phases [9]. Furthermore, recent constraints from a combination of type-Ia supernova (SNe Ia), baryon acoustic oscillation peak (BAO) and cosmic microwave background radiation (CMB) shift give  $n \in [-0.3, 0.3]$  and  $\alpha \in [1.3, 7.1]$  at 99.7% confidence level [8–12].

For gravity theory of the form (26) the positivity of the energy density (25) gives

$$m^{2n+2} + (2n+1)\alpha \geq 0. \quad (27)$$

Now taking into account the above dynamical systems constraint on  $n$  and  $\alpha$ , one has that there are real values for  $m$  such that (27) holds only for  $n$  in the interval  $n \in (-1, -0.5)$ , whose intersection with the above interval

<sup>2</sup> For studies of the interrelations between the energy conditions (on scales relevant for cosmology) and observational data see Ref. [26].

<sup>3</sup> Bampi and Zordan theorem was obtained originally in the framework of general relativity and extended to the context of  $f(R)$  gravity in the metric formalism in Ref. [19].

permitted by observations is empty. This makes clear that the Palatini  $f(R)$  gravity (26) does not admit Gödel geometry as a perfect-fluid solution with  $\rho > 0$ , and for the values of  $n$  and  $\alpha$  allowed by dynamical systems along with the above combination of observational data. In this sense, this theory remedies the causal pathology in the form of closed time-like curves which is allowed in general relativity.

### C. Single scalar field solutions

Since any perfect-fluid Gödel-type solution of Palatini  $f(R)$  gravity that is subject to the WEC condition  $\rho + p > 0$  is noncausal, the question as to whether other matter sources could generate Gödel-type causal solutions naturally arises. In this section we shall examine this question by considering another different matter source, namely a single scalar field  $\Phi(z)$ , whose energy momentum tensor is given by

$$T_{AB}^{(S)} = \Phi_{|A} \Phi_{|B} - \frac{1}{2} \eta_{AB} \Phi_{|M} \Phi_{|N} \eta^{MN}, \quad (28)$$

where a vertical bar denotes components of covariant derivatives relative to the local basis  $\theta^A = e_\alpha^{(A)} dx^\alpha$  [see Eqs. (12) and (13)]. Following Ref. [21] it is straightforward to show that a scalar field of the form  $\Phi(z) = \varepsilon z + \text{const}$  satisfies the scalar field equation  $\square \Phi = \eta^{AB} \nabla_A \nabla_B \Phi = 0$  for a constant amplitude  $\varepsilon$  of  $\Phi(z)$ . Thus, the non-vanishing components of energy-momentum tensor for this scalar field are

$$T_{00}^{(S)} = -T_{11}^{(S)} = -T_{22}^{(S)} = T_{33}^{(S)} = \frac{\varepsilon^2}{2}, \quad (29)$$

and the field equations (15) can be written in the form

$$(3\omega^2 - m^2)f_R + \frac{f}{2} = 0, \quad (30)$$

$$\omega^2 f_R - \frac{f}{2} = 0, \quad (31)$$

$$(m^2 - \omega^2)f_R - \frac{f}{2} = k^2 \varepsilon^2. \quad (32)$$

Equations (30) and (31) yield  $(4\omega^2 - m^2)f_R = 0$ , which leads to  $m^2 = 4\omega^2$  for any Palatini  $f(R)$  theories that satisfy the WEC condition  $f_R > 0$ . This gives rise to the unique class of Gödel-type solutions

$$m^2 = 4\omega^2, \quad (33)$$

$$f_R = \frac{\kappa^2 \varepsilon^2}{2\omega^2}, \quad (34)$$

$$f = k^2 \varepsilon^2, \quad (35)$$

where  $f$  and  $f_R$  are to be evaluated at  $R = 2(m^2 - \omega^2) = 3m^2/2$ . From equations (11) and (33) one clearly has that the critical radius  $r_c \rightarrow \infty$ . Hence, for this unique solution there is no violation of causality of Gödel type for any Palatini  $f(R)$  gravity with  $f_R > 0$ .

Finally, we note that the Palatini  $f(R)$  theory of form (26) permits this unique causal solution for the values of  $n$  and  $\alpha$  allowed by the above dynamical systems plus observational data analyses. Indeed, Eq. (26) along with Eqs. (33) – (35) give

$$m^2 = \frac{2}{3} \left[ \frac{\alpha}{2} (n+3) \right]^{1/(n+1)}, \quad (36)$$

which shows that  $m^2$  is independent of the amplitude of the scalar field. Thus, for  $n \in [-0.3, 0.3]$  and, for example,  $\alpha = 3.45$  (the best fit value found in [10]) one has  $m \in [2.5, 1.6]$ . For any other value of  $\alpha$  allowed by observations we obviously have a different range of values for  $m$  but again with no breakdown of causality of Gödel-type.

### V. FINAL REMARKS

A good deal of effort has recently gone into the study of the so-called  $f(R)$  gravity. This is motivated by the fact that these theories provide an alternative way to explain the late accelerating expansion of the Universe without invoking either dark energy matter component or the existence of an extra spatial dimension. If gravity is governed by a  $f(R)$  a number of issues should be reexamined in the  $f(R)$  framework. This includes, for example, solar system tests, a correct Newtonian limit, gravitational waves, black holes, four distinct phases in the evolution history of the Universe, and the breakdown of causality at a non-local scale.

The underlying space-time manifolds of  $f(R)$  gravity theories are assumed to be locally Lorentzian. Thus, in both formulation of  $f(R)$  gravity the causal structure of the space-time has the same local properties of the flat space-time of special relativity, and hence the causality principle is locally satisfied. The nonlocal question, however, is left open, and violation of causality can come about. The general relativity Gödel model [15] is the best known example of a cosmological solution of in which causality is violated at a nonlocal scale.

In this paper, we have examined Gödel-type models and the violation of causality problem in Palatini  $f(R)$  gravity generalizing the results of Refs. [19–21]. For physically well-motivated perfect-fluid matter source, we have shown that every solution with arbitrary  $\rho$  that satisfies the weak energy condition  $\rho + p \geq 0$  (or equivalently  $df/dR > 0$ ) is necessarily isometric to the Gödel geometry, making explicit that the violation of causality is a generic feature of Palatini  $f(R)$  gravity theories. This extends to the context of Palatini  $f(R)$  the Bampi-Zordan theorem [16] which was previously established in the context of Einstein's theory, and has been extended recently to the framework of  $f(R)$  in the metric formalism [19]. We have derived an expression for the critical radius  $r_c$  (beyond which the causality is violated) for an arbitrary Palatini  $f(R)$  theory that satisfies the WEC condition

$f_R \geq 0$  making apparent that the violation of causality depends upon both the  $f(R)$  gravity theory and the matter source components  $(\rho, p)$ .

We concretely studied Gödel-type perfect-fluid solutions in the specific  $f(R) = R - \alpha/R^n$  Palatini gravity theory. We showed that, for positive matter density (with  $\rho + p > 0$ ) and for  $\alpha$  and  $n$  within the interval permitted by the observational data, this theory does not admit Gödel geometry as a solution of its field equations. In this sense, this theory remedies the causal anomaly of Gödel type which is allowed in general relativity. We have also examined the violation of causality of Gödel type by considering a scalar field as a matter source. For this source we showed that any Palatini  $f(R)$  gravity gives rise to a

unique Gödel-type solution with no violation of causality.

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